# The Crystal Structure of a Monoclinic Form of Aluminium Metaphosphate, $\mathbf{A l}\left(\mathrm{PO}_{3}\right)_{3}$ 

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$\mathrm{Al}\left(\mathrm{PO}_{3}\right)_{3}$ crystallizes in the monoclinic space group Ic with $a=10 \cdot 423, b=18 \cdot 687, c=9 \cdot 222 \AA, \beta=$ $81.63^{\circ} ; Z=12$. In the [010] direction a subcell structure is present with a period of $b / 3$. The structure was solved from the Patterson function. It can be described as a network of Al octahedra and P tetrahedra. Infinite chains of tetrahedra run along [001] interconnected by octahedra, the repeat distance in the chain being six tetrahedra. The mean Al-O distance in the octahedra is $1.884 \AA$; the $\mathrm{P}-\mathrm{O}$ bonds which are linked to octahedra have a mean value of $1.483 \AA$; those connected to other tetrahedra are $1.577 \AA$.

## Introduction

Cubic $\mathrm{Al}\left(\mathrm{PO}_{3}\right)_{3}$ was first described by Hautefeuille \& Margottet (1883). This form has been investigated by Hendricks \& Wyckoff (1927) (space group), Pauling \& Sherman (1937) (structure determination), Wappler (1958) (refinement) and Kleber (1960) (growth forms). Other crystalline metaphosphates are reported by d'Yvoire (1960).

The observation of Hautefeuille \& Margottet that mixed crystals could be prepared with various quantities of $\mathrm{Cr}, \mathrm{Fe}$ and U , whose metaphosphates are reported as orthorhombic, was the motive for investigating the existence of forms other than cubic. Visser (1962) pointed out the resemblance of the powder diagram of the monoclinic form to that of monoclinic


Fig. 1. Numbering of the atoms.
$\mathrm{Fe}\left(\mathrm{PO}_{3}\right)_{3}$. The cell constants of the Fe compound are $a=13.137, b=19.083, c=9.395 \AA, \beta=126.95^{\circ}$, which suggest isomorphism. Here the crystal structure of monoclinic $\mathrm{Al}\left(\mathrm{PO}_{3}\right)_{3}$ is presented.

## Experimental

Crystals suitable for X-ray investigation were prepared by melting $\mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{HPO}_{3}$ in a Pt crucible. Cell constants were obtained from zero-level Weissenberg photographs calibrated with Al powder lines: $a=$ 10.423 (3), $b=18.687$ (2), $c=9.222$ (1) $\AA, \beta=81.63$ (1) ${ }^{\circ}$.

The space group is $I c$ or $I 2 / c$; the structure determination has shown the first to be correct. The crystals just sank in bromoform; the density therefore is about $2 \cdot 90$, whereas with $Z=12,2.96 \mathrm{~g} \mathrm{~cm}^{-3}$ is calculated.

Intensities were measured with a Nonius automatic single-crystal diffractometer AD3, operating in the $\omega$-scan mode. Two sets of intensities were collected, one with $\mathrm{Cu} K \alpha$ giving 967, the other with Mo $K \alpha$ radiation yielding 3752 observed non-zero intensities. In the data obtained with Cu radiation an intensity difference attributable to absorption could be observed between symmetry-equivalent reflexions; the other set did not show this. Therefore, the refinement has been carried out with the Mo measurements without absorption correction.

## Structure determination

From an inspection of the intensities it was apparent that a subcell structure was present. The reflexions with $k=3 n \pm 1$ were weak, suggesting a tripartitioning of $b$. Thus the structure determination was divided into two stages. First the average subcell structure was determined and after this the full crystal structure by deriving the deviations of the atoms from the average positions. The most probable space group is $I c$ since in this case the atoms of the 12 formula units in the cell can be placed at three sets of general positions.

A Patterson function was calculated from the 368 reflexions with $k=3 n$ from the set of $\mathrm{Cu} K \alpha$ data, transformed to a reduced cell. The orientation of the

Al octahedron was found from the $\mathrm{Al}-\mathrm{O}$ vectors on the sphere with radius $1.8 \AA$ around the origin. It can be expected that Al octahedra and P tetrahedra are connected in such a way that the angle Al-O-P is near $180^{\circ}$. As the $\mathrm{Al}-\mathrm{O}$ and $\mathrm{P}-\mathrm{O}$ lengths are 1.8 and $1.5 \AA$, it is assumed that the Al-P vectors will be found in the Patterson function on a sphere around the origin with a radius of about $3 \AA$.

When these vectors had been found, the Al-O-P triangle could be completed and the position of the O-P vectors located on the sphere with radius $1 \cdot 4 \AA$. From the knowledge of the orientation and connexion of octahedra and tetrahedra thus found, a model of the average structure was constructed.

The subcell structure, an average of three sets of positions, was refined by least squares with the $k=3 n$ subset of reflexions. Since the intensities with $k=3 n$ must also contain information about the deviations from the average positions, a difference synthesis was calculated with these reflexions. This difference map gave indications for offsetting six of the nine P atoms from their average positions. This proved to be sufficient for starting the refinement of the full structure.

It was not possible to refine all atoms with individual anisotropic temperature factors in an unrestricted fullmatrix refinement due to limitations in the program. Therefore in the final refinement the Al plus the P atoms and the O atoms were refined alternately, the thermal parameters of corresponding atoms in the three subcells


Fig. 2. $y z$ plane at $x=\frac{1}{2}$.

Table 1. Fractional coordinates $\left(\times 10^{4}\right)$ and thermal parameters $\left(\times 10^{5}\right)$
The standard deviation of each coordinate from refinement is $0.004,0.003$ and $0.007 \AA$ for $\mathrm{Al}, \mathrm{P}$ and O respectively.

|  | $x$ | $y$ | $z$ | $U_{11}$ | $U_{22}$ | $U_{33}$ | $2 U_{12}$ | $2 U_{23}$ | $2 U_{31}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Al}(1)$ | 5073 | 878 | 67 |  |  |  |  |  |  |
| $\mathrm{Al}(2)$ | 5053 | 4202 | 57 | 602 | 561 | 346 | -45 | 19 | -43 |
| $\mathrm{Al}(3)$ | 4867 | 7452 | -72 |  |  |  |  |  |  |
| $\mathrm{P}(1)$ | 6473 | 276 | 7010 |  |  |  |  |  |  |
| $\mathrm{P}(2)$ | 6310 | 3646 | 6889 | 506 | 337 | 241 | -42 | 49 | 53 |
| $\mathrm{P}(3)$ | 6289 | 6969 | 6840 |  |  |  |  |  |  |
| $\mathrm{P}(4)$ | 8756 | 310 | 3182 |  |  |  |  |  |  |
| $\mathrm{P}(5)$ | 8573 | 3635 | 3107 | 535 | 601 | 463 | 88 | -14 | -111 |
| $\mathrm{P}(6)$ | 8617 | 6986 | 3065 |  |  |  |  |  |  |
| $\mathrm{P}(7)$ | 7317 | 1486 | 4930 |  |  |  |  |  |  |
| P (8) | 7688 | 4848 | 5121 | 526 | 329 | 439 | -51 | -160 | 30 |
| $\mathrm{P}(9)$ | 7560 | 8183 | 5081 |  |  |  |  |  |  |
| $\mathrm{O}(1)$ | 179 | 2130 | 2994 |  |  |  |  |  |  |
| $\mathrm{O}(2)$ | 501 | 5353 | 3342 | 777 | 808 | 489 | 310 | -401 | 534 |
| $\mathrm{O}(3)$ | 295 | 8908 | 3071 |  |  |  |  |  |  |
| $\mathrm{O}(4)$ | 2244 | 1460 | 2522 |  |  |  |  |  |  |
| O(5) | 2884 | 5224 | 2449 | 859 | 1124 | 1194 | 571 | 184 | -720 |
| O (6) | 2246 | 8110 | 2562 |  |  |  |  |  |  |
| $\mathrm{O}(7)$ | 964 | 1632 | 484 |  |  |  |  |  |  |
| $\mathrm{O}(8)$ | 1262 | 4695 | 1000 | 1143 | 837 | 552 | -247 | -252 | -243 |
| $\mathrm{O}(9)$ | 885 | 8295 | 597 |  |  |  |  |  |  |
| $\mathrm{O}(10)$ | 1580 | 245 | 9706 |  |  |  |  |  |  |
| $\mathrm{O}(11)$ | 1365 | 3430 | 9521 | 946 | 1802 | 897 | 532 | 81 | $-160$ |
| $\mathrm{O}(12)$ | 1334 | 6959 | 9418 |  |  |  |  |  |  |
| $\bigcirc$ | 3729 | 255 | 674 |  |  |  |  |  |  |
| $\mathrm{O}(14)$ | 3475 | 3714 | 537 | 1234 | 157 | 839 | -759 | -241 | -143 |
| $\mathrm{O}(15)$ | 3467 | 6824 | 431 |  |  |  |  |  |  |
| $\mathrm{O}(16)$ | 3888 | 1438 | 9139 |  |  |  |  |  |  |
| $\mathrm{O}(17)$ | 4131 | 5003 | 9546 | 1146 | 956 | 610 | 758 | 745 | -9 |
| $\mathrm{O}(18)$ | 3714 | 8126 | 9321 |  |  |  |  |  |  |
| O (19) | 4625 | 2123 | 6777 |  |  |  |  |  |  |
| $\mathrm{O}(20)$ | 4857 | 5524 | 7034 | 1316 | 650 | 464 | 580 | -99 | 336 |
| O(21) | 4637 | 8637 | 6857 |  |  |  |  |  |  |
| $\mathrm{O}(22)$ | 2250 | 715 | 6379 |  |  |  |  |  |  |
| $\mathrm{O}(23)$ | 1584 | 3985 | 6194 | 1286 | 1022 | 1008 | 1217 | -436 | -168 |
| $\mathrm{O}(24)$ | 2157 | 7340 | 6434 |  |  |  |  |  |  |
| $\mathrm{O}(25)$ | 3340 | 589 | 3747 |  |  |  |  |  |  |
| $\mathrm{O}(26)$ | 2756 | 4055 | 3627 | 1474 | 649 | 747 | -178 | 755 | 398 |
| O(27) | 3376 | 7270 | 3857 |  |  |  |  |  |  |

being constrained to the same value. The refinement, in which a Cruickshank (1961) weighting scheme was used, was terminated at $R=0.057$ for 3752 observed reflexions. The final coordinates and thermal parameters are presented in Table 1.*

From the error estimates in the refinement, the errors in the coordinates can be set at about $0.004,0.003$ and $0.007 \AA$ for $\mathrm{Al}, \mathrm{P}$ and O respectively. The numbering of the atoms is given in Fig. 1.

* A list of structure factors has been deposited with the British Library Lending Division as Supplementary Publication No. SUP 31762 ( 17 pp., 1 microfiche). Copies may be obtained through The Executive Secretary, International Union of Crystallography, 13 White Friars, Chester CH1 1 NZ, England.


## Description of the structure

In Table 2 the bond distances and angles are presented. The estimated standard deviations from the leastsquares refinement are about $0 \cdot 008 \AA$ for the $\mathrm{Al}-\mathrm{O}$ and $\mathrm{P}-\mathrm{O}$ distances and $0.4^{\circ}$ for the $\mathrm{O}-\mathrm{Al}-\mathrm{O}$ and $\mathrm{O}-\mathrm{P}-\mathrm{O}$ angles. Since these figures are underestimates and should be multiplied by at least a factor of two, it may be concluded that Al octahedra probably do not deviate from regularity. The mean of the Al-O distances is 1.884 (5) $\AA$. The Al-O distances may be compared with those reported in other structures, e.g. wavellite, $\mathrm{Al}\left(\mathrm{PO}_{4}\right)_{2}(\mathrm{OH})_{3} .4 \frac{1}{2}-5 \mathrm{H}_{2} \mathrm{O}$ (Araki \& Zoltai, 1968); eosphorite, $\mathrm{AlPO}_{4}(\mathrm{Mn}, \mathrm{Fe})(\mathrm{OH})_{2} . \mathrm{H}_{2} \mathrm{O}$ (Hanson, 1960 ) ; and turquois, $\mathrm{CuAl}_{6}\left(\mathrm{PO}_{4}\right)_{4}(\mathrm{OH})_{8} \cdot 4 \mathrm{H}_{2} \mathrm{O}$ (Cid-

Table 2. Bond distances and angles

| $\mathrm{A}(1)-\mathrm{O}(2)$ | 1.87 |
| :---: | :---: |
| -O(8) | 1.93 |
| -O(11) | 1.88 |
| -O(13) | $1 \cdot 85$ |
| -O(16) | 1.91 |
| -O(21) | $1 \cdot 88$ |
| $\mathrm{P}(1)-\mathrm{O}(2)$ | 1.48 |
| -O(5) | 1.58 |
| -O(8) | $1 \cdot 47$ |
| -O(23) | 1.57 |
| $\mathrm{P}(4)-\mathrm{O}(5)$ | 1.57 |
| -O(17) | 1.48 |
| -O(20) | 1.50 |
| -O(26) | 1.59 |
| $\mathrm{P}(7)-\mathrm{O}(12)$ | 1.48 |
| -O(15) | 1.49 |
| -O(23) | 1.57 |
| -O(26) | 1.59 |
| $\mathrm{O}(2)-\mathrm{Al}(1)-\mathrm{O}(8)$ | 90 |
| -O(11) | 94 |
| -O(13) | 90 |
| -O(16) | 90 |
| -O(21) | 177 |
| $\mathrm{O}(8)-\quad-\mathrm{O}(11)$ | 91 |
| -O(13) | 91 |
| -O(16) | 180 |
| -O(21) | 88 |
| $\mathrm{O}(11)-\quad-\mathrm{O}(13)$ | 176 |
| -O(16) | 89 |
| -O(21) | 89 |
| $\mathrm{O}(13)-\quad-\mathrm{O}(16)$ | 88 |
| -O(21) | 88 |
| $\mathrm{O}(16)-\quad-\mathrm{O}(21)$ | 92 |
| $\mathrm{O}(2)-\mathrm{P}(1)-\mathrm{O}(5)$ | 110 |
| -O(8) | 117 |
| -O(23) | 108 |
| $\mathrm{O}(5)-\quad-\mathrm{O}(8)$ | 110 |
| -O(23) | 100 |
| $\mathrm{O}(8)-\quad-\mathrm{O}(23)$ | 111 |
| $\mathrm{O}(5)-\mathrm{P}(4)-\mathrm{O}(17)$ | 111 |
| -O(20) | 107 |
| -O(26) | 101 |
| $\mathrm{O}(17)-\quad-\mathrm{O}(20)$ | 116 |
| -O(26) | 108 |
| $\mathrm{O}(20)-\quad-\mathrm{O}(26)$ | 113 |
| $\mathrm{O}(12)-\mathrm{P}(7)-\mathrm{O}(15)$ | 118 |
| -O(23) | 107 |
| -O(26) | 106 |
| $\mathrm{O}(15)-\quad-\mathrm{O}(23)$ | 110 |
| -O(26) | 110 |
| $\mathrm{O}(23)-\quad-\mathrm{O}(26)$ | 105 |


| $\mathrm{Al}(2)-\mathrm{O}(3)$ | 1.89 |
| :---: | :---: |
| -O(7) | $1 \cdot 90$ |
| -O(10) | 1.89 |
| -O(14) | 1.88 |
| -O(17) | 1.88 |
| -O(20) | 1.88 |
| $\mathrm{P}(2)-\mathrm{O}(3)$ | 1.49 |
| -O(6) | 1.59 |
| -O(9) | 1.48 |
| -O(22) | 1.57 |
| $\mathrm{P}(5)-\mathrm{O}(4)$ | 1.57 |
| -O(18) | 1.49 |
| -O(21) | 1.48 |
| -O(25) | $1 \cdot 57$ |
| $\mathrm{P}(8)-\mathrm{O}(10)$ | 1.47 |
| -O(13) | 1.48 |
| -O(22) | 1.58 |
| -O(25) | 1.58 |
| $\mathrm{O}(3)-\mathrm{Al}(2)-\mathrm{O}(7)$ | 88 |
| -O(10) | 90 |
| -O(14) | 94 |
| -O(17) | 89 |
| -O(20) | 178 |
| $\mathrm{O}(7)-\quad-\mathrm{O}(10)$ | 93 |
| -O(14) | 90 |
| -O(17) | 177 |
| -O(20) | 90 |
| $\mathrm{O}(10)-\quad-\mathrm{O}(14)$ | 175 |
| -O(17) | 88 |
| -O(20) | 89 |
| $\mathrm{O}(14)-\quad-\mathrm{O}(17)$ | 89 |
| -O(20) | 86 |
| $\mathrm{O}(17)-\quad-\mathrm{O}(20)$ | 92 |
| $\mathrm{O}(3)-\mathrm{P}(2)-\mathrm{O}(6)$ | 109 |
| -O(9) | 118 |
| -O(22) | 108 |
| $\mathrm{O}(6)-\quad-\mathrm{O}(9)$ | 108 |
| -O(22) | 102 |
| $\mathrm{O}(9)-\quad-\mathrm{O}(22)$ | 110 |
| $\mathrm{O}(4)-\mathrm{P}(5)-\mathrm{O}(18)$ | 112 |
| -O(21) | 109 |
| -O(25) | 98 |
| $\mathrm{O}(18)-\quad-\mathrm{O}(21)$ | 116 |
| -O(25) | 109 |
| $\mathrm{O}(21)-\quad-\mathrm{O}(25)$ | 110 |
| $\mathrm{O}(10)-\mathrm{P}(8)-\mathrm{O}(13)$ | 118 |
| -O(22) | 112 |
| -O(25) | 108 |
| $\mathrm{O}(13)-\quad-\mathrm{O}(22)$ | 104 |
| -O(25) | 107 |
| $\mathrm{O}(22)-\quad-\mathrm{O}(25)$ | 107 |



Dresdner, 1965); in these structures the comparable Al-O distances range from 1.834 to $1.927 \AA$.

The P-O bonds connected to octahedra are about $0 \cdot 1 \AA$ shorter than the bonds to other tetrahedra. The mean of the short bonds is 1.483 (2) $\AA$, that of the longer ones 1.577 (2) $\AA$. The same is found in other metaphosphates, e.g. $\left(\mathrm{RbPO}_{3}\right)_{\infty}$ (Cruickshank, 1964); $\mathrm{CuLi}\left(\mathrm{PO}_{3}\right)_{3}$ (Laügt, Tordjman, Guitel \& Roudaut, 1972); $\mathrm{CuNa}_{2}\left(\mathrm{PO}_{3}\right)_{4}$ (Läugt, Tordjman, Guitel \& Bassi, 1972); $\left[\mathrm{Na}_{3} \mathrm{H}\left(\mathrm{PO}_{3}\right)_{4}\right]_{x}$ (Jost, 1968); $\left(\mathrm{NaPO}_{3}\right)_{x}$ (McAdam, Jost \& Beagley, 1968); and $\mathrm{Ba}_{2} \mathrm{~K}\left(\mathrm{PO}_{3}\right)_{s}$ (Martin, Tordjman \& Mitschler, 1972), where these distances fall in the ranges of $1.45-1 \cdot 51$ and $1.58-1.62$ $\AA$. The mean bond angle between the shorter bonds in the tetrahedra is $117.5(5)^{\circ}$. The angle between the longer bonds in the tetrahedra around $\mathrm{P}(7), \mathrm{P}(8)$ and $P(9)$ is different from those in the other tetrahedra; the respective means are $106 \cdot 2$ (5) and $100 \cdot 2$ (7) ${ }^{\circ}$. In the literature cited, values between 99 and $102^{\circ}$ are found. The structure consists of infinite chains of tetrahedra interconnected by Al octahedra. Each octahedron shares corners with six tetrahedra and two octahedra.

The manner in which the Al octahedra and P tetrahedra link together is made clear in Figs. 2, 3 and 4; the octahedra are marked I, II and III in order to discriminate between the three different octahedra stemming from the three subcell parts.

In Fig. 2 a view of the $y z$ plane around $x=\frac{1}{2}$ is given in which a network of alternating octahedra and tetrahedra exists. The connexion in the direction of $x$ is illustrated in Fig. 3, showing a picture of the $x y$ plane around $z=0$. Chains of alternating octahedra and tetrahedra run in the $x$ direction only. Along the $a$ glide plane at $y=\frac{3}{4}$ the chain is formed solely from type III octahedra. In the other two chains octahedra I and II alternate; the $a$ glide plane at $y=\frac{1}{4}$ interrelates these two. In a partial projection on the $y z$ plane in Fig. 4 the infinite chains of P tetrahedra running along [001] are indicated.

The repeat unit consists of six tetrahedra. Infinite chains of tetrahedra with a period of six tetrahedra are also found in $\mathrm{CuLi}\left(\mathrm{PO}_{3}\right)_{3}$ where they form a helix around a twofold screw axis. In contrast to this the chain in $\mathrm{Al}\left(\mathrm{PO}_{3}\right)_{3}$ describes a 'figure 8 ' when seen in projection along $z$. Al octahedra are linked to the chain, four tetrahedra forming a loop across an octahedron edge.

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## References

Araki, T. \& Zoltai, T. (1968). Z. Kristallogr. 127, 21-33. Cid-Dresdner, H. (1965). Z. Kristallogr. 121, 87-113. Cruickshank, D. W. J. (1961). In Computing Methods and the Phase Problem in X-ray Crystal Analysis. Oxford: Pergamon Press.


Fig. 3. $x y$ plane at $z=0$.


Fig. 4. Chains of tetrahedra in the $z$ direction.
Cruickshank, D. W. J. (1964). Acta Cryst. 17, 681-682.
Hanson, A. W. (1960). Acta Cryst. 13, 384-387.
Hautefeuille, P. \& Margottet, J. (1883). C. R. Acad. Sci. Paris, 96, 850-852.
Hendricks, S. B. \& Wyckoff, R. W. G. (1927). Amer. J. Sci. 13, 491.
Jost, K. H. (1968). Acta Cryst. B24, 992-996.
Kleber, W. (1960). Neues Jb. Miner. Abh. 94, 1060-1074.
Läugt, P. M., Tordjman, I., Guitel, J. C. \& Bassi, G. (1972). Acta Cryst. B28, 2721-2725.

Läugt, P. M., Tordjman, I., Guitel, J. C. \& Roudaut, M. (1972). Acta Cryst. B28, 2352-2358.

McAdam, A., Jost, K. H. \& Beagley, B. (1968). Acta Cryst. B24, 1621-1622.
Martin, C., Tordjman, I. \& Mitschler, A. (1972). Cryst. Struct. Commun. 1, 349-352.
Pauling, L. \& Sherman, J. (1937). Z. Kristallogr. 96, 481-487.
Visser, J. W. (1962). Private communication.
Wappler, G. (1958). Diplom-Arbeit Miner. Petr. Instituts der Humboldt Univ. 1-50.
D'Yvoire, F. (1960). C. R. Acad. Sci. Paris, 251, 2182-2184.

